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INTERPOLATION AND COORDINATE TRANSFORMATION PROGRAM PROVIDING ACCURATE LOCAL ANTENNA POINTING FROM PREDICTED SATELLITE POSITIONS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

This report describes a method of producing accurate pointing information for steerable antennas by utilizing local satellite position predictions and a small on-site digital computer. The local Cartesian coordinates of the satellite are generated at Goddard Space Flight Center's Computing Center and transmitted by teletype to the various antenna sites. The on-site computer performs an interpolation and coordinate transformation of these coordinates, incorporates various environmental considerations, and produces a drive tape for input to the antenna directing device. This system preserves completely the accuracy of the predictions generated at the Goddard Computing Center and performs the desired operations in 0.5 real time or less.

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INTRODUCTION

The National Aeronautics and Space Administration is constructing a network of large-aperture steerable antennas to meet the data acquisition requirements of future satellite programs. Figures 1 and 2 show two typical antennas recently installed at NASA data acquisition facilities. These large aperature steerable antennas operate at high frequencies and consequently have very narrow beamwidths: those shown have a beamwidth of approximately 0.5 degree for frequencies now in use. Because of the narrow beamwidths, accurate horizon-to-horizon satellite pointing information is necessary in order to acquire the satellite initially and to back up the auto-track system.

Experience in providing pointing information for large aperture steerable antennas was gained during Project ECHO in which 1-second antenna drive tapes were generated at the GSFC Computing Center and transmitted by teletype to the various antenna stations. Transmitting such data (azimuth angle, elevation angle, and time) for each second of time resulted in performing the operations in approximately 2.4 real time; i.e., it required about 24 minutes to transmit data for a 10-minute satellite pass. This gave excellent pointing accuracy, but the teletype transmission was too time consuming. Therefore, it was necessary to develop a method that would decrease the teletype transmission time but preserve the accuracy as it exists at the GSFC computer.

Dr. J. W. Siry of GSFC has recommended that antenna drive tapes be produced at the antenna sites rather than at GSFC. By transmitting prediction data less frequently, and interpolating with an on-site computer, the amount of data would be reduced and hence the time needed to transmit the data would also be reduced. However, to preserve the accuracy of the GSFC Computing Center, the predictional data should not consist of angles; this is because of the technical difficulty caused by extreme rates of change of azimuth angle at zenith; and a similar condition holds for X-angle (see Figure 4) at the north and south points on the horizon.

The method recommended by Dr. Siry for producing antenna drive tapes on site involves an interpolation and coordinate transformation of antenna pointing predictions generated at the GSFC
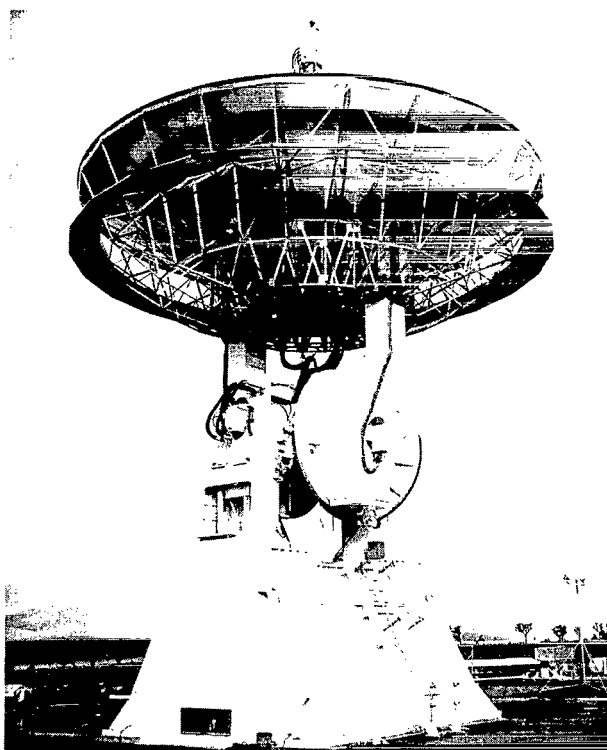
Figure 1—The 40-foot NASA antenna with X-Y angle mount at Mojave, California.
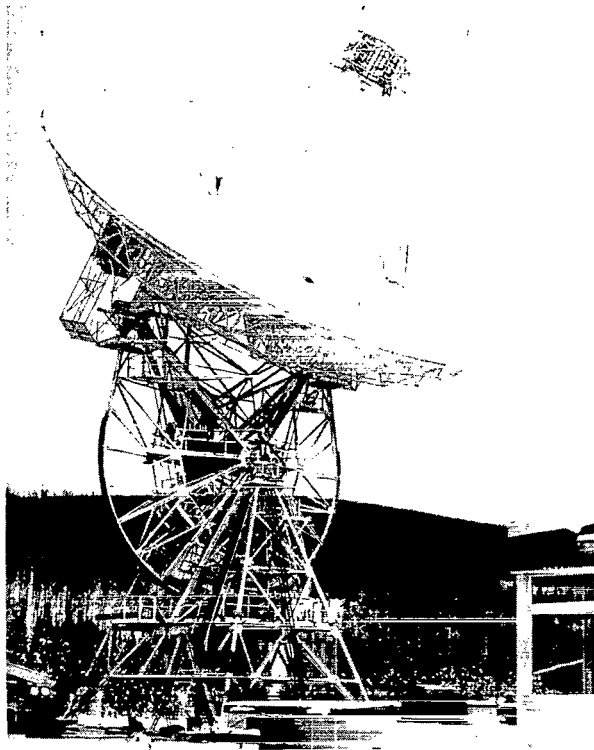

Figure 2—The 85-foot NASA antenna with X-Y angle mount at Fairbanks, Alaska.

Figure 3—The Packard Bell 250 Computer.

Computing Center and teletyped from GSFC to the antenna stations. The on-site computer at the antenna station interpolates in these coordinates and transforms the resulting positions to the coordinate system required by the antenna directing devices. Following these recommendations, an Interpolation and Coordinate Transformation Program (I&CT) was written. Because present schedules indicate that several data acquisition facility antennas will be in continuous operation (24 hours a day), it was considered necessary that the on-site computer be capable of performing the desired operations in 0.5 real time or less. The I&CT Program was written for the Packard Bell 250 Computer (Figure 3) which was determined to be the least expensive computer capable of meeting this requirement.

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ANTENNA POINTING PREDICTIONS

The antenna pointing predictions, consisting of the satellite coordinates in a local topocentric Cartesian coordinate system, are specified at regular intervals (usually 1-minute intervals). The directions of the coordinate axes in this system are toward the local east (E), north (N), and vertical (V), respectively. The satellite position vector components are expressed in hundredths of kilometers. The local topocentric Cartesian coordinates (E, N, V) are teletyped in the format described in Table 1 to the antenna station.

The on-site computer at the antenna station performs a sliding 5th-degree polynomial interpolation of the E, N, and V coordinates at 1/60 of the input interval, transforms these coordinates to pointing angles, and punches out an antenna drive tape. The format for a typical line is shown in Table 2. The antenna drive tapes are compatible with teletype tapes (5/8 inch, 5-level Baudot coded tape). Therefore, the antenna pointing information can be transmitted directly from the Goddard Computing Center in case of malfunction of the on-site computer.

POLYNOMIAL FIT AND INTERPOLATION

The I&CT Program fits a 5th degree polynomial in time (t) to six consecutive input E, N, or V points; computes 59 equally spaced interpolation points between the 3rd and 4th input; then slides (advances) the fit one point and repeats the procedure for the next six consecutive input points. This is accomplished in the following manner. The interpolation polynomial has the form:

$$E(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 , \qquad a_i = \text{constants};$$

$$N(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 , \qquad b_i = \text{constants};$$

$$V(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 , \qquad c_i = \text{constants}.$$

In matrix notation, the fit to the coordinates may be expressed

$$
\begin{bmatrix}
1 & t_1 & t_1^2 & t_1^3 & t_1^4 & t_1^5 \\
1 & t_2 & t_2^2 & t_2^3 & t_2^4 & t_2^5 \\
1 & t_3 & t_3^2 & t_3^3 & t_3^4 & t_3^5 \\
1 & t_4 & t_4^2 & t_4^3 & t_4^4 & t_4^5 \\
1 & t_5 & t_5^2 & t_5^3 & t_5^4 & t_5^5 \\
1 & t_6 & t_6^2 & t_6^3 & t_6^4 & t_6^5
\end{bmatrix}
\begin{bmatrix}
a_0 & b_0 & c_0 \\
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3 \\
a_4 & b_4 & c_4 \\
a_5 & b_5 & c_5
\end{bmatrix}
=
\begin{bmatrix}
E_i & N_i & V_i \\
E_{i+1} & N_{i+1} & V_{i+1} \\
E_{i+2} & N_{i+2} & V_{i+2} \\
E_{i+3} & N_{i+3} & V_{i+3} \\
E_{i+4} & N_{i+4} & V_{i+4} \\
E_{i+5} & N_{i+5} & V_{i+5}
\end{bmatrix}
$$

Table 1

Teletype Transmission Format of Local Topocentric Cartesian
Coordinates (E, N, V).

Line Number on Teletype Printout	Printout	Explanation
1	Local coordinate predictions for SA ID	SA ID is a 5-digit satellite identification. Example: 58022 equals 58 beta 2.
2	YYDDD HHMMSS	Beginning date and time (GMT) for predictions.
3	YYDDD HHMMSS	Ending date and time (GMT) for predictions.
4	SSSSSS	6-character station code.
5	Z	Z indicates to the computer the beginning of a pass.
6	YYDDD HHMMSS DTM	1. Lines 6 through N consist of predictions for one pass and will repeat for each pass in the prediction. 2. Date and time (GMT) of first prediction line of pass followed by DTM, which equals time interval in minutes between the coordinates.
7 through N-1	SEEEEEEEESNNNNNNNNNSVVVVVVVV PPP	(1) Local E, N, and V coordinates in hundredths of kilometers. (2) PPP is a parity check equal to the decimal sum of the E, N, and V digits plus those digits assigned to S and spaces (no digits are assigned to CR and LF). (3) S is the sign (space for positive and minus sign for negative). (4) Local E is local east (positive east, negative west). (5) Local N is local north (positive north, negative south). (6) Local V is local vertical (positive up, negative down).
N	LL	Total number of position vectors (E, N, V) in one pass.
Last Line	ZZ	ZZ indicates the end of predictions.

or, in more convenient notation,

$$TA = C ,$$

$$A = T^{-1} C .$$ (1)

The interpolation between the 3rd and 4th input may be expressed

$$
\begin{bmatrix}
1 & t_3 & t_3^2 & t_3^3 & t_3^4 & t_3^5 \\
1 & t_{3_1} & t_{3_1}^2 & t_{3_1}^3 & t_{3_1}^4 & t_{3_1}^5 \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
1 & t_{3_{59}} & t_{3_{59}}^2 & t_{3_{59}}^3 & t_{3_{59}}^4 & t_{3_{59}}^5 \\
1 & t_4 & t_4^2 & t_4^3 & t_4^4 & t_4^5
\end{bmatrix}
\begin{bmatrix}
a_0 & b_0 & c_0 \\
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3 \\
a_4 & b_4 & c_4 \\
a_5 & b_5 & c_5
\end{bmatrix}
=
\begin{bmatrix}
E_{(i+2)} & N_{(i+2)} & V_{(i+2)} \\
E_{(i+2)_1} & N_{(i+2)_1} & V_{(i+2)_1} \\
. & . & . \\
. & . & . \\
E_{(i+2)_{59}} & N_{(i+2)_{59}} & V_{(i+2)_{59}} \\
E_{(i+3)} & N_{(i+3)} & V_{(i+3)}
\end{bmatrix}
$$

Table 2

Antenna Drive Tape Format.

Character	Function
1	Carriage return
2	Line feed
3	*Sign (X-angle)
4	Tens of degrees (X angle)
5	Units of degrees (X angle)
6	Tenths of degrees (X angle)
7	Hundredths of degrees (X angle)
8	Thousands of degrees (X angle in 0.002 degree increments)
9	Space
10	*Sign (Y angle)
11	Tens of degrees (Y angle)
12	Units of degrees (Y angle)
13	Tenths of degrees (Y angle)
14	Hundredths of degrees (Y angle)
15	Thousands of degrees (Y angle in 0.002 degree increments)
16	Space
17	Figures
18	Tens of hours
19	Units of hours
20	Tens of minutes
21	Units of minutes
22	Tens of seconds
23	Units of seconds

*The digit 1 indicates a positive sign, and zero indicates a negative sign.

or

$$T_I A = C_I , \qquad (2)$$

where sub-subscripts indicate the 59 interpolated values of dependent and independent variables. Combining Equations 1 and 2 gives

$$C_I = T_I T^{-1} C = QC . \qquad (3)$$

Since the input coordinates are always equally spaced in time and the interpolation interval is constant, the matrix Q is constant. Therefore, in order to slide the fit by one point and obtain the next 59 interpolated values, it is necessary only to change C (increase the subscripts by 1) and perform the matrix multiplication indicated in Equation 3.

COORDINATE TRANSFORMATION

The coordinate transformation from the E, N, V coordinate system to the $X-Y$ or $AZ-EL$ pointing angles is derived from Figure 4.

$$X = \tan^{-1} \frac{E}{V} ,$$

$$Y = \tan^{-1} \frac{N}{\left(E^2 + V^2\right)^{1/2}} ,$$

$$AZ = \tan^{-1} \frac{E}{N} ,$$

$$EL = \tan^{-1} \frac{V}{\left(E^2 + N^2\right)^{1/2}} .$$
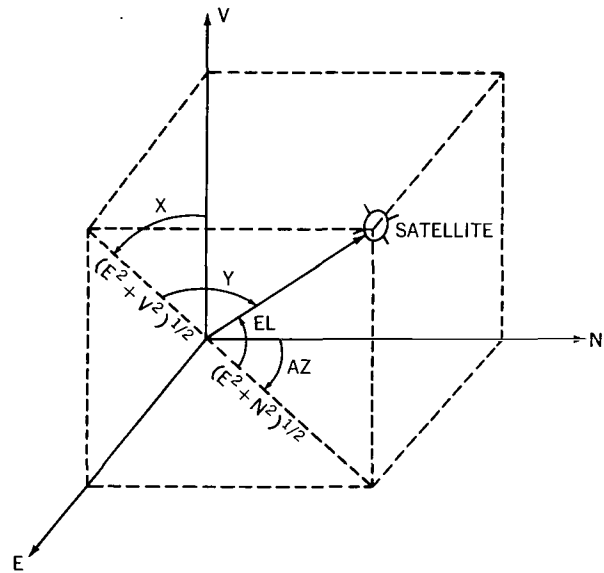

Figure 4—Coordinate transformation.

ANTENNA LIMITS

In order that the antenna not be driven into its physical limits by the antenna drive tape, pointing angles outside the antenna limits are not punched on the tape. The I&CT Program searches for those pointing angles at which the satellite enters and leaves the antenna field of view (approximately 1/2 degree above the antenna prelimit switch settings). These entering and leaving angles are punched respectively at the beginning and end of the drive tape for two minutes duration. Under drive tape control, the antenna will wait for the satellite at the position where it is expected to rise above the limits, follow it while it is above these limits, and stop rotating when the satellite goes below these limits.

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The antenna limits are represented in terms of elevation angle. For AZ-EL antennas, a typical elevation angle limit is 5 degrees completely around the horizon. For X-Y antennas, the elevation angle limit is a function of the azimuth angle. The dotted line in Figure 5 is representative of operational limits of large X-Y antennas. (Only the S-E quadrant is shown in Figure 5. The limits are symmetric with respect to the N-S and E-W planes.) It can be seen that a given satellite pass could enter and leave the dotted line limits several times. Since this situation presents difficulties to the I&CT Program in its search for those angles which are within the field of view, it is desirable to represent the antenna limits by a smooth curve for which a satellite can enter and leave the field of view only once on a given pass. Such a curve (I&CT Program Limits) is also shown. The program performs the following computations to determine when the satellite is above the antenna limits (refer to Figure 5):
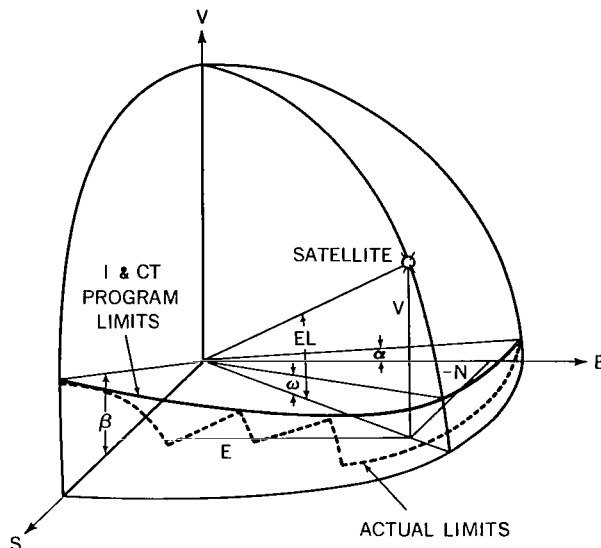

Figure 5—Antenna limits.

Satellite elevation (EL):

$$\tan \text{EL} = \frac{V}{\left(E^2 + N^2\right)^{1/2}} .$$

Elevation of smooth limit curve (ω) at satellite azimuth:

$$\tan \omega = \frac{A|E| + B|N|}{\left(E^2 + N^2\right)^{1/2}} ,$$

where $A = \tan \alpha$ and $B = \tan \beta$ (α and β are constants for a given antenna).

The numerators of the expressions for $\tan \text{EL}$ and $\tan \omega$ are now compared to determine whether the satellite elevation is greater than the antenna limit. If $V - \left(A|E| + B|N|\right) > 0$, the satellite is in the field of view. If $V - \left(A|E| + B|N|\right) \leq 0$, the satellite is out of the field of view.

ATMOSPHERIC REFRACTION CORRECTION

In addition to interpolation and coordinate transformation, the I&CT Program applies a correction for the atmospheric refraction of the radio signal. The refraction correction R is applied

to the zenith angle and computed with the following formula:

$$R = \left[\frac{79P}{T} + \frac{(38 \times 10^4)e}{T^2}\right]\left[\tan Z - \frac{295}{(90° - Z + 1.1°)^3}\right] \times 10^{-6} \text{ radians },$$

where

$$T = \text{temperature in degrees Kelvin,}$$
$$P = \text{pressure in millibars,}$$
$$e = \text{water vapor pressure in millibars,}$$
$$Z = \text{zenith angle in degrees.}$$

The term $295/(90° - Z + 1.1°)^3$ in the above formula is an empirical addition to the usual theoretical expression. This term makes it possible to obtain agreement in R to within one second of arc with the values tabulated* for zenith angles down to 85 degrees for the expected range of temperatures.

In the application of the atmospheric refraction correction for AZ-EL drive tapes, the elevation angle is increased by the angle R as shown in Figure 6. For X-Y drive tapes, a ΔV correction is added to the V component of the satellite (see Figure 6) before computation of the X and Y angles as functions of E, N, and V. The ΔV correction is computed as a function of R:

Figure 6—Atmospheric refraction correction.

$$R = \frac{\Delta V \cos EL}{r};$$

thus

$$\Delta V = \frac{rR}{\cos EL} = \frac{r^2 R}{\left(E^2 + N^2\right)^{1/2}},$$
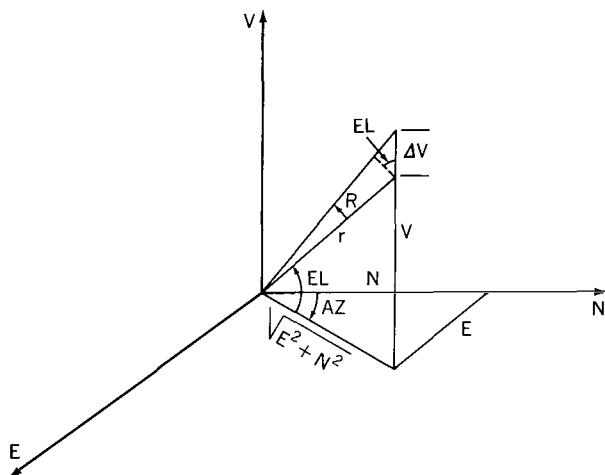
where

$$r^2 = E^2 + N^2 + V^2.$$

The X and Y angles are computed with the formulas given earlier, where now
V = V + ΔV.

IONOSPHERIC REFRACTION CORRECTION

No refraction correction has been made for the ionosphere in this program. Since the high frequencies which result in the narrow beamwidths are less affected by the ionosphere than by the

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*Haskinson, A. J., and Duerksen, J. A., "Manual of Geodetic Astronomy," U. S. Government Printing Office, Washington, D. C., 1952, pp. 174-175, Tables V, VI, and VII.

atmosphere, ionospheric refraction has not been considered of great importance. However, a study is presently being conducted to determine if a correction for ionospheric refraction at both low and high frequencies should be included.

ANTENNA MOUNT MISALIGNMENT AND CALIBRATION CORRECTION

The success of this program depends on the antenna pointing accuracy. Not only must the antenna pointing predictions be accurate, but the physical antenna must be aligned properly. Misalignment of the axes or the effects of gravity on the feed support may result in pointing inaccuracies. A pointing error of 0.25 degree will result in a 3 decibel loss in received signal level.

If the antenna's inherent pointing errors are consistent and can be measured, a calibration error correction function can be applied to the I&CT Program. An extensive effort is in progress to determine the pointing accuracy of the antennas for which the I&CT Program was written.

As of this writing, calibration functions have been derived for the antennas shown in Figures 1 and 2. They have improved the pointing data at the time of calibration. However, too few calibrations have been performed to determine the consistency of the antenna's inherent pointing errors.

Where an axis misalignment or a non-orthogonal condition exists, it is normally more economical to correct these design errors by inserting a mathematical compensation in the I&CT Program than to physically realign the antenna.

DATA CAPACITY

The I&CT Program will accept a maximum of 77 input coordinates (sets of E, N, or V) at any input interval from 1 to 60 minutes. The input interval must be an integral number of minutes. Since 77 input coordinates will be accepted by the program, and each input may have a 60 minute interval, the data capacity of the I&CT Program covers passes from 1 minute to 77 hours in duration, and should be adequate for any future needs.

CONCLUSIONS

This system preserves completely the accuracy of the predictions generated at the GSFC Computing Center; provides a standard prediction data format which will not need revision each time the GSFC orbit programs are modified; is applicable to either azimuth-elevation, equatorial, or X-Y angle antennas, and decreases the teletype transmission time by a factor of 40 with the interpolation from minutes to seconds of time by the small on-site digital computer.

Although the teletype transmission time has been greatly reduced, it has been more time consuming in some cases than was anticipated. This has been mainly due to the many high priority interruptions encountered during the 1-1/2 hours required to send one week's predictions for a

typical satellite (three other stations are on the same transmission line to Alaska). Each time an interruption occurred, a hand reconstruction of the ENV tape was required before it could be accepted by the on-site PB-250 computer. It is desired to reduce the teletype transmission time further and thereby decrease the probability of interruptions.

An investigation is in progress to determine the feasibility of reducing teletype transmission time by sending Brouwer "mean" elements rather than ENV minute vectors for generating the antenna drive tapes on site.

(Manuscript received September 18, 1963)

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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